Punching Shear Design to EC2 (EN 1992-1-1-2004 (E))


For normal concrete flat slabs 200mm deep or greater.

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**Required information.**

Certain information is needed before a design can be undertaken:

- The size/shape of the loaded area (column, pile or wall).
- The characteristic compressive cylinder strength of concrete ($f_{ck}$).
- The mean ratio of tension reinforcement in both directions in a width of the column + 3d each side (column under: top reinforcement, column over: bottom reinforcement).
- slab reinforcement drawing may be necessary, if this information isn't given.
- The slab thickness.
- Top and bottom cover to the reinforcement.
- Design value of the applied shear force $V_{Ed}$ (Ultimate load: factored)

It is assumed that:

That any loads given by the Project Engineer have been factored using the EC load factors (not from BS8110).

The enhancement factor ($\beta$) to be used is as recommended in 6.4.3. Figure 6.21N, unless the Project Engineers advise otherwise.

The concrete slab is not constructed using lightweight aggregate.

Alternatively, the Engineer can provide the enhancement factor $\beta$. We should make sure that loads given are only the slab loads and do not include the column above.

- The slab condition: i.e. Internal, edge or corner conditions, plus dimensions of the slab edge from the face of the column.

Other consideration:

- Position and size of any hole/s in the slab within 6d from the edge of the supporting column, pile, wall.
- Any changes in slab thickness, steps in level or movement joints local to the column/pile – a general layout drawing may be necessary.

It is assumed that:

For internal columns 50% area of the top reinforcement should be provided placed in a equal width to the sum of 0.125 times the panel each side of the column to resist the full negative moment.  

For internal columns there will be a minimum of bottom two bars in each orthogonal direction provided and this reinforcement should pass through the column.

Similarly, the edge and corner columns conditions, the slab should be reinforcement to comply with clause 9.4.2 (1)
1. Enhancement factor (\( \beta \))

If the structure is unbraced or if adjacent spans differ by more than 25%, \( \beta \) should be calculated, refer to section 6.4.3 (3), (4) & (5).
(It is assumed that the Project Engineer will provide this information where applicable).

The enhancement factor \( \beta \) is taken from figure 6.21N

- Internal column \( \beta = 1.15 \)
- Edge Column \( \beta = 1.4 \)
- Corner Column \( \beta = 1.5 \)

The \( \beta \) factor for External/Re-entrant corner column might be taken as 1.275.
Average between internal and edge column: \( \beta = (1.15 + 1.4) / 2 = 1.275 \)

2. Shear at the Column Face/Perimeter (\( u_0 \))

a. Effective depth (\( d \))

\[
d_{eff} = (d_y + d_z) / 2
\]

Flat slabs/piled ground slabs (column/pile supporting under the slab):
The effective depth (\( d_{eff} \)) is taken as the average depth of the top reinforcement to underside of the Slab

Raft foundation/Transfer slab (slab supporting column from above):
The effective depth (\( d_{eff} \)) is taken as the average depth of the bottom reinforcement to the top of the Slab.
b. Perimeter of loaded area. \((u_0)\) 6.4.5 (3)

Internal column \(u_0 = 2c_1 + 2c_2\)

Edge column \(u_0 = c_2 + 3d\) or \(u_0 = 2c_1 + c_2\) (Smallest value of the two equations)

Corner column \(u_0 = 3d\) or \(u_0 = c_1 + c_2\) (Smallest value of the two equations)

Note: holes within 6d of the face of the loaded area will reduce \(u_0\) accordingly. 6.4.2 (3) also see section 13 Holes/Penetrations in the slab

c. Design value of the applied shear stress \((v_{Ed 0})\) at the face of the column.

\[ v_{Ed 0} = \beta \frac{V_{Ed}}{u_0 d} \]

Where

\[ v = 0.6 \left(1 - \frac{f_{ck}}{250}\right) \]

hence \[ v_{Rd,max} = 0.3 f_{cd} \left(1 - \frac{f_{ck}}{250}\right) \]

Where \(f_{cd}\) the value of the design compressive strength of concrete.

\[ f_{cd} = \alpha_{cc} f_{ck} / y_c \]

\(f_{ck}\) is the characteristic compressive cylinder strength of concrete at 28 days, this can be found from table 3.1 in the code.

\[ \alpha_{cc} = 1 \] coefficient for long term effects

\[ y_c = 1.5 \] partial factor for material for ULS

\[ V_{Ed 0} \leq V_{Rd,max} \]

When \(V_{Ed 0}\) is greater than \(V_{Rd,max}\) the slab depth(h) or the column size must be increased
3. **Control Perimeter ($u_1$)**

Basic control perimeter at 2d from the loaded area (column or pile) $u_1$

**Internal column**

$$u_1 = 2(c_1 + c_2) + 2\pi(2d)$$

**Edge column**

$$u_1 = 2c_1 + c_2 + \pi(2d)$$

**Corner column**

$$u_1 = c_1 + c_2 + \frac{\pi(2d)}{2}$$

*Note:* holes within 6d of the face of the loaded area will reduce $u_1$ accordingly.

6.4.2 (3) also see section 13

Holes/penetrations in the slab
4. Punching Shear Resistance at the Control Perimeter ($u_1$) without Reinforcement.

a. Punching Shear Resistance ($v_{Rdc}$) at 2d

$$v_{Rdc} = C_{Rdc} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \geq (v_{min} + k_1 \cdot \sigma_{cp})$$

(post-tensioned design) 6.4.4 (6.47)

where

$$k_1 = 0.1$$

NA to BS EN 1992-1-1-2004 6.4.4 (1)

$$\sigma_{cp} = (\sigma_{cy} + \sigma_{cz})/2$$

$\sigma_{cy} \& \sigma_{cz}$ are the normal concrete stress in the critical section in y- and z- directions (MPa, positive if compression)

$$\sigma_{cy} = N_{ed.y} / A_{cy} \& \sigma_{cz} = N_{ed.z} / A_{cz}$$

6.4.4 (1)

$N_{ed.y}$ & $N_{ed.z}$ Are the longitudinal forces across the full bay for internal columns and the longitudinal force across the control section for edge columns. The force may be from a load or pre-stressing action.

$A_c$ is the area of concrete according to the definition of $N_{ed}$.

$$v_{Rdc} = C_{Rdc} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \geq v_{min}$$

(non-post or pre-tensioned design) 6.4.4 (6.47)

where

$$k = 1+\sqrt{\frac{200}{d}}$$

less than or equal to 2 6.4.4 (6.47)

$$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}}$$

mean reinforcement ratio, $\leq 0.02$ 6.4.4 (6.47)

$\rho_{ly}$ and $\rho_{lz}$ are the mean ratio of tension reinforcement in both directions (width of column + 3d each side).

$$\rho_{ly} = A_{sl.y} / (bd_y) \& \rho_{lz} = A_{sl.z} / (bd_z)$$

where $b = 1000$ mm

$$C_{Rdc} = 0.18/\gamma_c$$

NA to BS EN 1992-1-1-2004 6.4.4 (1)

$$\gamma_c = 1.5$$

partial factor for material for ULS 2.4.2.4 table 2.1N

$$v_{min} = 0.035k^{3/2}f_{ck}^{1/2}$$

NA to BS EN 1992-1-1-2004 6.4.4 (1) and stated in 6.2.2 (6.3N)

Transposed to :

$$v_{Rdc} = \left(0.18/\gamma_c\right) \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \geq v_{min} \& \gamma_c = 1.5$$

(Concise Eurocode 2, June 2006)

or

$$v_{Rdc} = 0.12 \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \geq v_{min}$$

(non post-tensioned design)

$$v_{Rdc} = 0.12 \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + 0.1 \cdot \sigma_{cp} \geq (v_{min} + 0.1 \cdot \sigma_{cp})$$

(post-tensioned design)

b. Design value of the maximum shear stress at the control perimeter $u_1$

$$v_{Ed} = \beta \cdot V_{Ed} / (u_1d)$$

6.4.3 (3) 6.38

c. Design value of the maximum punching shear resistance

$$V_{Ed} < V_{Rdc}$$

punching shear reinforcement is not required 6.4.3 (2b)

$$V_{Ed} > 2V_{Rdc}$$

Exceeds the maximum limit allowed in the UK National Annex

Increase the slab properties.. i.e. Top reinforcement, Depth, etc.. 6.4.5 (3)

Where

$$v_{Ed}$$ Actual stress at the perimeter $u_1$

$V_{Rdc}$ Punching shear resistance at $u_1$ (without reinforcement).
5. Punching Shear Resistance ($v_{Rd.cs}$) at the Control Perimeter ($u_1$) with Reinforcement.

a. Punching Shear Resistance ($v_{Rd.cs}$)

$$v_{Rd.cs} = 0.75 \, v_{Rd.c} + 1.5 \, (d / s_r) \, A_{sw} \, f_{ywd.ef} \, (1 / (u_1 \, d)) \, \sin \alpha$$

Where

$$v_{Rd.c} \leq v_{Ed.1}$$

$$v_{Rd.c} = 0.12 \, k \, (100 \, \rho_i \, f_{ck})^{1/3}$$

$v_{Ed.1}$ = The actual stress at the perimeter at $u_1$ is $\beta \, V_{Ed} / (u_1 \, d)$

$A_{sw}$ Area of one perimeter of shear reinforcement around the column. (to be greater than $A_{sw.min}$ given below)

$s_r$ Radial spacing of the perimeter reinforcement.

$f_{ywd.ef}$ The effective design strength of the punching reinforcement.

$$f_{ywd.ef} = 250 + 0.25 \, d \quad \text{less than or equal to} \quad f_{ywd} = (f_y / 1.15)$$

$\alpha$ The angle between the shear reinforcement (studs at 90° = $\sin 90° = 1$)

Transposed to:

$$v_{Rd.cs} = 0.75 \, v_{Rd.c} + 1.5 \, (d / s_r) \, A_{sw} \, f_{ywd.ef} \, (1 / (u_1 \, d)) \quad \text{(vertical shear reinforcement)}$$

$$v_{Rd.cs} - 0.75 \, v_{Rd.c} = 1.5 \, (d / s_r) \, A_{sw} \, f_{ywd.ef} \, (1 / (u_1 \, d))$$

$$(v_{Rd.cs} - 0.75 \, v_{Rd.c}) / 1.5 \, f_{ywd.ef} = (d / s_r) \, A_{sw} / (u_1 \, d)$$

$$(v_{Rd.cs} - 0.75 \, v_{Rd.c}) / 1.5 \, f_{ywd.ef} = A_{sw} / (u_1 \, s_r)$$

$V_{Ed.1} = V_{Rd.cs}$

$$A_{sw} = (V_{Ed.1} - 0.75 \, v_{Rd.c}) \, u_1 \, s_r / (1.5 \, f_{ywd.ef}) \quad \text{(per perimeter) \quad (Concise Eurocode 2, June 2006)}$$

or

$$A_{sw} / s_r = (V_{Ed.1} - 0.75 \, v_{Rd.c}) \, u_1 / (1.5 \, f_{ywd.ef}) \quad \text{(as RCC Spreadsheet)}$$

b. Minimum area of a Link/Stud

$$A_{sw.min} = (1.5 \, \sin \alpha + \cos \alpha) / (s_r \, s_t) \quad \text{greater or equal to} \quad 0.08 \, \sqrt{(f_{ck}) / f_{yk}}$$

$s_t$ Spacing of the reinforcement in the tangential direction. & $s_r$ as above.

$f_{ck}$ is the characteristic compressive cylinder strength of concrete at 28 days, this can be found from table 3.1

$f_{yk}$ is the characteristic tensile strength of reinforcement 500 N/mm²

$\alpha$ The angle between the shear reinforcement (studs at $sin 90° = 1 & cos 90° = 0$)

Transposed to:

$$A_{sw.min} \times 1.5 / (s_r \, s_t) \geq 0.08 \, \sqrt{(f_{ck}) / f_{yk}} \quad \text{(IStuctE Manual Eurocode 2, Sept 2006)}$$

$$A_{sw.min} = 0.08 \, (s_r \, s_t) \, \sqrt{(f_{ck}) / 1.5 \, f_{yk}}$$

$$A_{sw.min} / s_r = 0.08 \, s_t \, \sqrt{(f_{ck}) / (1.5 \, f_{yk})} \quad \text{(as RCC Spreadsheet)}$$

Reinforcement to be detailed in accordance of 9.4.3.
4. Control perimeter where shear reinforcement is not required ($U_{out}$ or $U_{out,ef}$)

The outermost perimeter of shear reinforcement should be placed at a distance not greater than $kd$ within $U_{out}$ or $U_{out,ef}$.

\[ K = 1.5 \text{ unless the perimeter } U_{out} \text{ or } U_{out,ef} \text{ is less than } 3d \text{ from the face of loaded area (column/pile). In this case the reinforcement should be placed in the zone } 0.3d \text{ to } 1.5d \text{ from the face of the column.} \]

NA to BS EN 1992-1-1-2004 6.4.5 (4)

There should be a minimum of two perimeters of reinforcement.

9.4.3 (1)

The spacing of the reinforcement perimeters should not exceed 0.75d.

9.4.3 (1)

The first stud is placed not less than 0.3d from the face of the support.

Similarly, the distance between the face of a support or circumference of a loaded area and the nearest shear reinforcement taken into account in the design should not exceed $d/2$.

9.4.3 (4)

The spacing of reinforcement around a perimeter should not exceed 1.5d within the control perimeter (2d from the loaded area), and should not exceed 2d for perimeters outside the control perimeter, where that part of the perimeter is assumed to contribute to the shear capacity.

Reference 6.4.5 (4) figure 6.22

\[ Kd = 1.5d \]

\[ U_{out} \text{ or } U_{out,ef} = \beta V_{Ed} / (V_{Rd,c} d) \]

6.4.5 (4)

The shape of the perimeter $U_{out}$ or $U_{out,ef}$ will fluctuate in accordance to the arrangement of the shear reinforcement and spacing limitations.

With reference to 6.4.5 (4) figure 6.22
5. Shearail Layout – Spiral/Circular Pattern - Square Column.
a. Calculating the position of the perimeter $U_{\text{out}}$ or $U_{\text{out},\text{ef}}$

Ideally the corner rails on a spiral layout pattern are set at 30° & 60° giving equal lengths around $U_{\text{out}}$ for ‘$e$’.

Using a pre-fabricated rail with fixed values for $x_1$ and $x_2$ reduces the complication of manufacture process and fixing on site.

For this reason, we reconsider the setting out for $x_1$ and $x_2$ to be equal, this in turn will increase the length of $e$ to $e_1$ and to $e_2$, slightly increase the length of $U_{\text{out}}$.

This increase is ignored on the basis that the larger perimeter will have an increased load capacity and is therefore a worst-case is being considered.

*Note: $g_2 =$ half the column size, to a maximum of 0.75$d$*

*Denotes: The diagram indicates maximum spacing values in terms of ‘$d$’ (effective depth). When the stud spacing is set to a maximum of 0.75$d$ and $g_2$ is also set at the maximum of 0.75$d$, this allows for a site location tolerance between the studs around the perimeter within the maximum stud spacing of 0.15$d$ at 2$d$.

Provide intermediate corner rails when more than 4 perimeters of reinforcement are required (more than four studs on a rail).
When the column size is smaller than 1.5d then

Note: \( g_2 = \text{half the column size, to a maximum of 0.75d} \)

Assuming the slightly worst case that all the sides are equal

\[ U_{out} = 12e \quad \text{hence} \quad e = \frac{U_{out}}{12} \]

\[ g = e \left( \frac{\sin 75^\circ}{\sin 30^\circ} \right) = 1.932e \]

\[ g_1 = g - 1.553d - g_2 \]

When the column size is larger than 1.5d than

\[ e_1 = c_1 - 2g_2 \]

\[ e_2 = c_2 - 2g_2 \]

\[ U_{out} = 12e + 2e_1 + 2e_2 \]

hence \( e = \frac{(U_{out} - 2e_1 - 2e_2)}{12} \)

\[ g = e \left( \frac{\sin 75^\circ}{\sin 30^\circ} \right) = 1.932e \]

\[ g_1 = g - 1.553d - g_2 \]
b. General rules for a Spiral/Circular Pattern

position the last stud from column face at \( g_1 \), provide a minimum of 2 studs on a rail.

\[
U_{out} = \beta \frac{V_{Ed}}{(n_{Rd.c} \cdot d)}
\]

Internal condition – Square/rectangular Column

When the column size is equal to \( 2g_2 \)  
\[ e = \frac{U_{out}}{12} \]

otherwise  
\[ e = \frac{(U_{out} - 2e_1 - 2e_2)}{12} \]

Edge condition – Square/rectangular Column

\[ e_1 = c_1 - g_2 \quad \text{&} \quad e_2 = c_2 - 2g_2 \]

When the column size \( c_2 \) is equal to or less than \( 2g_2 \)  
\[ U_{out} = 6e + 2e_1 \quad \text{therefore} \quad e = \frac{(U_{out} - 2e_1)}{6} \]

otherwise  
\[ e = \frac{(U_{out} - 2e_1 - e_2)}{6} \]

Corner condition – Square/rectangular Column

\[ e_1 = c_1 - g_2 \quad \text{&} \quad e_2 = c_2 - g_2 \]

\[ U_{out} = 3e + e_1 + e_2 \quad \text{therefore} \quad e = \frac{(U_{out} - e_1 - e_2)}{3} \]

for all conditions :

\[ g = e \left( \sin 75^\circ / \sin 30^\circ \right) = 1.932 \ e \]

\[ g_1 = g - 1.553 \ d - g_2 \]

stud spacing = \( (g_1 - 0.5d) / (\text{number of stud on a rail} - 1) \)  

(less than or equal to 0.75d)
a. Calculating the position of the perimeter $U_{out}$ or $U_{out,ef}$

Ideally the rails spiral around the column are at an angle between 30° to 45° giving equal lengths around $U_{out}$ for 'e', resulting with a layout of 8 to 12 main rails.

The same pre-fabricated rail can be used throughout the layout.

b. General rules for a Spiral/Circular Pattern

1. Position the last stud from column face = $g_1$
2. Minimum number of studs on a rail is 2.

$$U_{out} = \beta \frac{V_{Ed}}{(\nu_{Rd,c} \cdot d)}$$

The number of rail spurs depends on the spacing rules inside 2d perimeter (less than or equal to 1.5d) and on the last stud spacing (less than or equal to 2d).

As $U_{out}$ is a polygon of equal sides: the number of sides = the number of main rail spurs.

$$e_3 \leq 2d \quad \text{(without intermediate rails)} \quad \text{or} \quad e_3 \leq 4d \quad \text{(with intermediate rails)}$$

$$e = \frac{U_{out}}{\text{No. of spurs}}$$

$$e_3 = e - 2e_2 \quad \text{where} \quad e_2 = 1.5d \cdot \tan F^\circ$$

Therefore try 8 spurs (as a standard layout) increasing the number of spurs until $e_3 \leq 2d$ or $e_3 \leq 4d$ with intermediate rails.

$$g = \left(\frac{e}{\sin E^\circ}\right) \times \sin \left(\frac{(180^\circ - E^\circ)}{2}\right)$$

$$g_1 = g - g_2 - g_3 \quad \text{where} \quad g_2 = \text{column diameter} / 2 \quad g_3 = 1.5d / \cos F^\circ$$

The 1st stud from column face = 0.5d

$$\text{stud spacing} = \frac{(g_1 - 0.5d)}{(\text{number of stud on a rail -1})}$$

(less than or equal to 0.75d)
9. Shearail Layout – Cruciform Pattern. (Note: Magenta coloured studs are not used in the design).

a. Calculating the position of the perimeter $u_{out}$ or $u_{out,ef}$

position the first stud from the column face = 0.3d

position the last stud from the column face = $g_1 + 0.3d$

minimum number of studs on a rail is 2

stud spacing = $g_1 / \text{number of stud on a rail}$ (less than or equal to 0.75d)

Stud spacing at last 2d ($s_2$) = 1.932 \((2 \times \text{stud spacing})\) (less than or equal to 1.5d)

$U_{out} = \beta \frac{V_{Ed}}{\nu_{frd.c} d}$

Note: the value $U_{out,ef}$ will not increase after more than three perimeters of reinforcement.
\[ u_{\text{out. ef}}^c = (0.6d + c_1) + (3d\pi) / 4 + 2d \]
\[ = 2.6d + c_1 + (3d\pi) / 4 \]

Similarly, for \( c_2 \) column side:
\[ u_{\text{out. ef}}^c = 2.6d + c_2 + (3d\pi) / 4 \]

**Internal condition** (based on a minimum of three perimeters of reinforcement/studs)
\[ u_{\text{out}} = 2(2.6d + c_1 + (3d\pi) / 4) + 2(2.6d + c_2 + (3d\pi) / 4) \]
\[ = 5.2d + 2c_1 + (3d\pi) / 2 + 5.2d + 2c_2 + (3d\pi) / 2 \]
\[ = 10.4d + 2c_1 + 2c_2 + 3d\pi \]

**Edge condition** (based on a minimum of three perimeters of reinforcement/studs)
\[ u_{\text{out}} = 2(1.3d + c_1 + (3d\pi) / 8) + 2.6d + c_2 + (3d\pi) / 4 \]
\[ = 2.6d + 2c_1 + (3d\pi) / 4 + 2.6d + c_2 + (3d\pi) / 4 \]
\[ = 5.2d + 2c_1 + c_2 + (3d\pi) / 2 \]

**Corner condition** (based on a minimum of three perimeters of reinforcement/studs)
\[ u_{\text{out}} = 1.3d + c_1 + (3d\pi) / 8 + 1.3d + c_2 + (3d\pi) / 8 \]
\[ = 2.6d + c_1 + c_2 + (3d\pi) / 4 \]
10. Example calculation – Internal condition

Data
Slab depth \( h = 300 \) mm
Load \( V_{ED} = 900 \) kN
Cover = 30 mm (top and bottom)
Reinforcement T1 & T2 = H16 @ 150c/c
Compressive strength of concrete \( f_{ck} = 30 \text{MPa} \)

a. Spiral/Circular Pattern – 300mm Square Column

Internal column \( \beta = 1.15 \) (unless advised otherwise by the Project Engineer).

\[
\begin{align*}
\text{d} & = 300 - 30 - 16/2 - 16/2 = 254 \text{ mm} \\
\text{dy} & = 300 - 30 - 16/2 = 262 \text{ mm} \\
\text{dz} & = 300 - 30 - 16 - 16/2 = 246 \text{ mm}
\end{align*}
\]

Shear at the column face

\[
\begin{align*}
\text{\( u_0 \)} & = 4 \times 300 \text{ (note: any holes within 6d need to be allowed for)} = 1200 \text{ mm} \\
\beta \ V_{ED} & = 1.15 \times 900 = 1035 \text{ kN} \\
\text{\( v_{ED_0} \)} & = \beta \ V_{Ed} / (u_0 \ d) = 1035 \times 1000 / (1200 \times 254) = 3.396 \text{ MPa} \\
\text{f}_{cd} & = \alpha_{cc} \ f_{ck} / \gamma_c = 1 \times 30 / 1.5 = 20 \text{ MPa} \\
\text{\( v_{Rd_{\text{max}}} \)} & = 0.3 \ f_{cd} \ (1 - (f_{ck} / 250)) = 0.3 \times 20 \ (1 - (30 / 250)) = 5.28 \text{ MPa}
\end{align*}
\]

\[
\begin{align*}
\text{Check if } \text{\( v_{ED_0} \)} & \leq \text{\( v_{Rd_{\text{max}}} \)} \\
3.396 \text{ MPa} & \leq 5.28 \text{ MPa} \text{ OK!}
\end{align*}
\]

Shear at control perimeter at 2d

\[
\begin{align*}
\text{\( u_1 \)} & = 2(c_1 + c_2) + 2\pi \ (2d) = 4 \times 300 + 2 \times \pi \times (2 \times 254) = 4392 \text{ mm}
\end{align*}
\]

- Shear at the control perimeter without reinforcement

\[
\begin{align*}
\text{\( C_{Rd_{c}} \)} & = 0.18 / \gamma_c = 0.18 / 1.5 = 0.12 \\
\text{k} & = 1 + \sqrt{(200 / d)} = 1 + \sqrt{(200 / 254)} = 1.887 \leq 2 \\
\text{\( v_{min} \)} & = 0.035 \text{k}^{3/2} \text{\( f_{ck} \)}^{1/2} = 0.035 \times (1.887)^{3/2} \times (30)^{1/2} = 0.497 \text{ MPa} \\
\text{\( v_{Ed_1} \)} & = \beta \ V_{Ed} / (u_1 \ d) = 1035 \times 1000 / (4392 \times 254) = 0.928 \text{ MPa}
\end{align*}
\]

Consider reinforcement over 300 + 6 x 254 = 1.824m width in both directions from centre of column.

Using H16 @ 150c/c both directions = 1340.41 mm²/m T1 & T2

\[
\begin{align*}
\text{\( p_i = \sqrt{(A_{sly} / \ (b \ d_y)) \times A_{slz} / (b \ d_z)} = \sqrt{(1340.41 / (1000 \times 262)) \times 1340.41 / (1000 \times 246))} = 0.00528 < 0.02 \]
\text{\( v_{Rd_{c}} \)} & = C_{Rd_{c}} \ k (100 \ \rho_i \ \text{\( f_{ck} \)}^{1/3} = 0.12 \times 1.887 \times (100 \times 0.00528 \times 30)^{1/3} = 0.569 \text{ MPa}
\end{align*}
\]

\[
\begin{align*}
\text{Check if } \text{\( v_{Rd_{c}} \)} & \geq \text{\( v_{min} \)} \\
0.569 & > 0.497 \text{ Ok! (use largest value)} \\
\text{Check if } \text{\( V_{ED_1} \)} & < \text{\( v_{Rd_{c}} \)} \\
0.928 & > 0.569 \text{ Shear reinforcement required} \\
\text{Check if } \text{\( V_{ED} \)} & \leq 2 \text{\( v_{Rd_{c}} \)} \\
0.928 & \leq 1.138 \text{ Below 2\text{\( v_{Rd_{c}} \)} limitation}
\end{align*}
\]
\[ U_{out\ required} = \beta \frac{V_{Ed}}{(v_{Rd,c} d)} = 1.15 \times 900 \times 1000 / (0.5688 \times 254) = 7164\text{mm} \]

(note: any holes within 6d need to be allowed for)

**Shearail Layout – Spiral/Circular Pattern**

0.75d = 190.5 mm
300 / 2 = 150 mm therefore: position rail central about column face in each direction

hence \( g_2 = 150 \text{ mm} \)

\[ e = \frac{U_{out}}{12} = \frac{7164}{12} = 597 \text{ mm} \]

\[ g = 1.932e = 1.932 \times 597 = 1153.4 \text{ mm} \]

\[ g_1 = g - 1.553d - g_2 = 1153.4 - 1.553 \times 254 - 150 = 609 \text{ mm} \]

1\(^{st}\) stud from column face = 0.5d = 127 mm

say 125 mm

distance of 1\(^{st}\) to last stud = 609 - 125 = 484 = 3 @165 mm < 0.75d = 190.5mm

Maximum stud distances on perimeter \( (s_{1.5}) = 1.5d = 381\text{mm} \) & \( (s_{last}) = 2.0d = 508\text{mm} \)

\[ v_{Ed,1} = \beta \frac{V_{Ed}}{(u_1 d)} = 1035 \times 1000 / (4392 \times 254) = 0.9278 \text{ MPa} \]

\[ f_{ywd,ef} = 250 + 0.25 d = 250 + 0.25 \times 254 = 313.5 \text{ N/mm}^2 \]

\[ f_{ywd} = (f_y / 1.15) = 500 / 1.15 = 434.78 \text{ N/mm}^2 > 313.5 \text{ ok!} \]

--- from earlier setting out: spacing rules are less then, 1.5d within 2d & 2d at the forth stud.

Check this example for confirmation only:

Distance to 1\(^{st}\) stud = \( (150 / \cos 30^\circ) + 125 \)

= 298.2 mm

length to 3\(^{rd}\) stud from column face = 2 x 165 + 298.2

= 628.2 mm

length to last stud from column face = 3 x 165 + 298.2

= 793.2 mm

\[ s_{1.5} = \sqrt{(628.2^2 + 628.2^2 - 2 \times 628.2 \times 628.2 \times \cos 30^\circ)} = 325.5 \text{ mm <1.5d} \]

\[ s_{last} = \sqrt{(793.2^2 + 793.2^2 - 2 \times 793.2 \times 793.2 \times \cos 30^\circ)} = 410.6 \text{ mm <2.0d} \]
Shear at the control perimeter with reinforcement

\[ A_{sw,\text{min}} = 0.08 \, s_t \, s_r \, \sqrt{f_{ck}} / (1.5 \, f_{yk}) = 0.08 \times 165 \times 410.6 \sqrt{30} / (1.5 \times 500) = 39.6 \, \text{mm}^2 \]

\[ A_{sw} = (v_{Ed} - 0.75 \, v_{Rd.c}) \, u_1 \, s_r / (1.5 \, f_{ywd,eff} \, \text{rail no.}) \]

\[ A_{sw} = (0.928 - 0.75 \times 0.569) \times 4392 \times 165 / (1.5 \times 313.5 \times 12) = 64.4 \, \text{mm}^2 \rightarrow \text{stud dia} = 10 \, \text{mm} \, (A = 78.54 \, \text{mm}^2) \]

Provide 12 No 10-4-240-745 (942 mm²). Spacing: 125/165/165/165/125 48 Studs total

Rail Layout
b. Spiral/Circular Pattern. – 300mm dia internal Circular column

All data as per 300mm square column.

Internal column $\beta = 1.15$ (unless advised otherwise by the Project Engineer).

$d = 300 - 30 - 16/2 - 16/2 = 254\text{mm}$

d$y = 300 - 30 - 16/2 = 262\text{mm}$

d$z = 300 - 30 - 16 - 16/2 = 246\text{mm}$

**Shear at the column face**

$u_0 = \pi \times 300$ (note: any holes within 6d need to be allowed for) = 943 mm

$\beta V_{ED} = 1.15 \times 900 = 1035 \text{kN}$

$v_{ED \, 0} = \beta V_{Ed} / (u_0 d) = 1035 \times 1000 / (943 \times 254) = 4.324 \text{ MPa}$

$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c = 1 \times 30 / 1.5 = 20 \text{ MPa}$

$v_{Rd,\, max} = 0.3 f_{cd} (1 - (f_{ck} / 250)) = 0.3 \times 20 (1 - (30 / 250)) = 5.28 \text{ MPa}$ or

check if $v_{ED \, 0} \leq v_{Rd,\, max}$ 4.324 MPa $\leq$ 5.28 MPa OK!

**Shear at control perimeter at 2d**

$u_1 = \pi ((4 \times 254) + 300)$ (note: any holes within 6d need to be allowed for) = 4134 mm

Shear at the control perimeter without reinforcement

$C_{Rd,\, c} = 0.18 \gamma_c = 0.18 / 1.5 = 0.12$

$k = 1 + \sqrt{(200 / d)} = 1 + \sqrt{(200 / 254)} = 1.887 \leq 2$

$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} = 0.035 \times (1.887)^{3/2} \times (30)^{1/2} = 0.497 \text{ MPa}$

$v_{Ed \, 1} = \beta V_{Ed} / (u_1 d) = 1035 \times 1000 / (4134 \times 254) = 0.986 \text{ MPa}$

Consider reinforcement over $300 + 6 \times 254 = 1.824\text{m}$ width in both directions from centre of column.

Using H16 @ 150c/c both directions = 1340.41 mm$^2$/m

$\rho_l = \sqrt{(A_{uly} / (b \, d_y) \times A_{als} / (bd_z))} = \sqrt{(1340.41 / 262 \times 1340.41 / 246) / 1000} = 0.00528 < 0.02$

$v_{Rd,\, c} = C_{Rd,\, c} k (100 \rho_l f_{ck})^{1/3} = 0.12 \times 1.887 (100 \times 0.00528 \times 30)^{1/3} = 0.569 \text{ MPa}$

check if $v_{Rd,\, c} \geq v_{min}$ 0.569 $\geq$ 0.497 Ok! (use largest value)

check if $V_{ED \, 1} < v_{Rd,\, c}$ 0.986 $> 0.569$ Shear reinforcement required

check if $V_{ED \, 1} \leq 2v_{Rd,\, c}$ 0.928 $\leq$ 1.138 Below $2v_{Rd,\, c}$ limitation

$U_{out \, required} = \beta V_{Ed} / (v_{Rd,\, c} d) = 1.15 \times 900 \times 1000 / (0.569 \times 254) = 7164\text{mm}$

(note: any holes within 6d need to be allowed for)
Shearail Layout – Spiral/Circular Pattern

\[ U_{\text{out\ required}} = 7164 \text{mm} \]

Try: 8 spurs

\[ e = 7164 / 8 = 895.5 \text{ mm} \]
\[ e_2 = 1.5 \times 254 \times \tan 22.5^\circ = 157.8 \text{ mm} \quad F = 180^\circ / 8 = 22.5^\circ \]
\[ e_3 = 895.5 - 157.8 \times 2 = 579.9 \text{ mm} \quad 2d (508) > 579.9 < 4d (1016) \]

*therefore 8 spurs with intermediate spacer rails...* \[ 579.9 / 2 = 289.9 < 2d \text{ ok!} \]

Try 10 spurs

\[ e = 7164 / 10 = 716.4 \text{ mm} \]
\[ e_2 = 1.5 \times 254 \times \tan 18^\circ = 123.8 \text{ mm} \quad F = 180^\circ / 10 = 18^\circ \]
\[ e_3 = 716.4 - 123.8 \times 2 = 468.8 \text{ mm} \quad 468.8 < 2d \text{ (508 mm)} \]

*therefore: use: 10 spurs without intermediate spacer rails*

\[ E = 360 / 10 = 36^\circ \]

\[ g = (716.4 / \sin 36^\circ) \times \sin ((180^\circ - 36^\circ) / 2) = 1159.14 \text{ mm} \quad \text{say 1160 mm} \]
\[ g_2 = 300 / 2 = 150 \text{ mm} \quad g_3 = (1.5 \times 254) / \cos 18^\circ = 400.6 \text{ mm} \quad \text{say 400 mm} \]
\[ g_1 = 1160 - 150 - 400 = 610 \text{ mm} \]

1st stud from column face = 0.5d = 127 mm \quad \text{say 125 mm}

Stud spacing = (610 - 125) = 485 mm = 3 @ 165 mm < .75d = 190.5 mm

Maximum stud distances on perimeter within 2d (s_{13}) = 1.5d = 381 mm

Distance to 3rd stud r_3 = 150 + 125 + 165 + 165 = 605 mm

\[ s_{13} = 2 r_3 \sin (E / 2) = 2 \times 605 \times \sin (36 / 2) = 373.9 \text{ mm} < 1.5d \text{ (381 mm)} \]

\[ v_{\text{Ed,1}} = \beta V_{\text{Ed}} / (u_1 d) = 1035 \times 1000 / (4134 \times 254) = 0.986 \text{ MPa} \]
\[ f_{\text{ywd,ef}} = 250 + 0.25 d = 250 + 0.25 \times 254 = 313.5 \text{ N/mm}^2 \]
\[ f_{\text{ywd}} = (f_y / 1.15) = 500 / 1.15 = 434.78 \text{ N/mm}^2 > 313.5 \text{ ok!} \]

- Shear at the control perimeter with reinforcement

\[ A_{\text{sw,min}} = 0.08 s_t s_r \sqrt{f_{cd}} / (1.5 f_{pk}) = 0.08 \times 165 \times 476 / (1.5 \times 500) = 45.9 \text{ mm}^2 \]
\[ A_{\text{sw}} = (v_{\text{Ed,1}} - 0.75 v_{\text{Ros,uc}}) u_1 s_t / (1.5 f_{\text{ywd,ef}} \text{ rail no.}) \]
\[ A_{\text{sw}} = (0.986 - 0.75 \times 0.569) \times 4134 \times 165 / (1.5 \times 313.5 \times 10) = 81.1 \text{ mm}^2 \quad \text{→ stud dia = 12 mm (A = 113.09 mm}^2) \]

Provide 10 No 12-4-240-745 (1131 mm²) Spacing: 125/165/165/165/125 40 Studs total
**Rail Layout**

10 No 12-4-240-745 (1131 mm²) Spacing: 125/165/165/165/125  40 Studs total
**Cruciform Pattern.**

*General rule: If more than 3 perimeters of studs are required then the Cruciform pattern is normally unsuitable.*

\[ U_{\text{out,required}} = 7164 \text{ mm} \]

\[ U_{\text{out}} = 10.4 \times 254 + 2 \times 300 + 2 \times 300 + 3 \times 254 \times \pi = 6235.5 \text{ mm} \]

\[ U_{\text{out,ef}} = \text{from scaled diagram} = 1556.46 \times 4 = 6225.84 \text{ mm} \]

(first stud rounded down to 75 mm)

**Cruciform pattern is unsuitable**

\[ U_{\text{out,ef}} < U_{\text{out,required}} \]

**Rail Layout**

40 Studs total
11. Example calculation – Edge condition

Data
Slab depth \( h = 300 \) mm
Load \( V_{\text{ED}} = 450 \) kN
Cover = 30 mm (top and bottom)
Reinforcement T1 & T2 = H16 @ 150c/c
Compressive strength of concrete \( f_{ck} = 30 \) MPa

a. Spiral/Circular Pattern.

Edge column \( \beta = 1.4 \) (unless advised otherwise by the Project Engineer).

\[
\begin{align*}
d &= 300 - 30 - 16/2 - 16/2 = 254 \text{ mm} \\
d_y &= 300 - 30 - 16/2 = 262 \text{ mm} \\
d_z &= 300 - 30 - 16 - 16/2 = 246 \text{ mm}
\end{align*}
\]

Shear at the column face

\( u_0 = 300 + 2 \times 300 \) (1.5\( d = 381 \) therefore use \( C_1 \)) = 900 mm

\( \beta V_{\text{ED}} = 1.4 \times 450 = 630 \text{ kN} \)

\( v_{\text{ED}0} = \beta V_{\text{Ed}} / (u_0d) = 630 \times 1000 / (900 \times 254) = 2.756 \text{ MPa} \)

\( f_{cd} = \alpha_{cc} f_{ck} / \gamma_c = 1 \times 30 / 1.5 = 20 \text{ MPa} \)

\( v_{\text{Rd,max}} = 0.3 f_{cd} (1 - (f_{ck} / 250)) = 0.3 \times 20 \times (1 - (30 / 250)) = 5.28 \text{ MPa} \)

\[ \text{check if } v_{\text{ED}0} \leq v_{\text{Rd,max}} \]

Shear control perimeter at 2\( d \)

\( u_1 = 3 \times 300 + \pi \times (2 \times 254) = 2496 \text{ mm} \)

Shear at the control perimeter without reinforcement

\( C_{\text{Rd,c}} = 0.18 / \gamma_c = 0.18 / 1.5 = 0.12 \)

\( k = 1 + \sqrt{(200 / d)} = 1 + \sqrt{(200 / 254)} = 1.887 \leq 2 \)

\( v_{\text{min}} = 0.035 k^{3/2} f_{ck}^{1/2} = 0.035 \times (1.887)^{3/2} \times (30)^{1/2} = 0.497 \text{ MPa} \)

\( v_{\text{Ed}} = \beta V_{\text{Ed}} / (u_1d) = 630 \times 1000 / (2496 \times 254) = 0.994 \text{ MPa} \)

Consider reinforcement over 300 + 6 x 254 = 1.824m width in both directions from centre of column.

Using H16 @ 150c/c both directions = 1340.41 mm²/m \( \rho_l = \sqrt{(A_{\text{sh}} / (b \ d_y) \times A_{\text{shz}} / (b \ d_z))} = \sqrt{(1340.41 / (1000 \times 262) \times 1340.41 / (1000 \times 246))} = 0.00528 < 0.02 \)

\( v_{\text{Rd,c}} = C_{\text{Rd,c}} k (100 \rho_l \ f_{ck})^{1/3} = 0.12 \times 1.887 \times (100 \times 0.00528 \times 30)^{1/3} = 0.569 \text{ MPa} \)

\[ \text{check } v_{\text{Rd,c}} \geq v_{\text{min}} \] 0.569 ≥ 0.497 Ok! (use largest value)

\[ \text{check } V_{\text{ED}} \leq v_{\text{Rd,c}} \] 0.994 > 0.569 Shear reinforcement required

\[ \text{check } V_{\text{ED}} \leq 2v_{\text{Rd,c}} \] 0.928 < 1.138 Below 2\( v_{\text{Rd,c}} \) limitation

\( U_{\text{out required}} = \beta V_{\text{Ed}} / (V_{\text{Rd,c}} \ d) = 1.4 \times 450 \times 1000 / (0.569 \times 254) = 4362 \text{ mm} \)

(note: any holes within 6d need to be allowed for)
Shearail Layout – Spiral/Circular Pattern

$0.75d = 190.5 \text{ mm}$

$300/2 = 150 \text{ mm}$  therefore: position rail central about column face in each direction  

hence $g_2 = 150 \text{ mm}$

$U_{\text{out required}} = 4362 \text{ mm}$

$e_1 = 300 - 150 = 150 \text{ mm}$

$e = (4362 - 2 \times 150) / 6 = 677 \text{ mm}$

$g = 1.932e = 1.932 \times 677 = 1308 \text{ mm}$

$g_1 = g - 1.553d - g_2 = 1308 - 1.553 \times 254 - 150 = 764 \text{ mm}$

$1^{st}$ stud from column face $= 0.5d = 127 \text{ mm}$  say $125 \text{ mm}$

distance of $1^{st}$ to last stud $= 764 - 125 = 639$ say $4 @ 160 \text{ mm} < .75d = 190.5\text{mm}$

Maximum stud distances on perimeter $(s_{1.5}) = 1.5d = 381\text{ mm} \quad \& \quad (s_{\text{last}}) = 2.0d = 508\text{ mm}$

$v_{E1d} = \beta \frac{V_{Ed}}{u_1 d} = 630 \times 1000 / (2496 \times 254) = 0.9938 \text{ MPa}$

$f_{ywd,ef} = 250 + 0.25 d = 250 + 0.25 \times 254 = 313.5 \text{ N/mm}^2$

$f_{ywd} = \frac{f_y}{1.15} = 500 / 1.15 = 434.78 \text{ N/mm}^2 > 313.5 \text{ ok!}$

--- from earlier setting out: spacing rules are less then, $1.5d$ within $2d$ & $2d$ at the forth stud.

Check this example for confirmation only:

Distance to $1^{st}$ stud = $(150 / \cos 30^\circ) + 125 = 298.2 \text{ mm}$

length to $3^{rd}$ stud from column face = $2 \times 160 + 298.2 = 618.2 \text{ mm}$

length to last stud from column face = $4 \times 160 + 298.2 = 938.2 \text{ mm}$

$s_{1.5} = \sqrt{(618.2^2 + 618.2^2 - 2 \times 618.2 \times 618.2 \times \cos 30^\circ)} = 320.0 \text{ mm} < 1.5d$

$s_{\text{last}} = \sqrt{(938.2^2 + 938.2^2 - 2 \times 938.2 \times 938.2 \times \cos 30^\circ)} = 485.6 \text{ mm} < 2.0d$
Shear at the control perimeter with reinforcement

\[ A_{sw,\min} = 0.08 \, s_t \, s_r \, \sqrt{f_{ck}} / (1.5 \, f_{yk}) = 0.08 \times 165 \times 410.6 \sqrt{30} / (1.5 \times 500) = 39.6 \, \text{mm}^2 \]

\[ A_{sw} = (v_{Ed} - 0.75 \, v_{Rd,c}) \, u_1 \, s_r / (1.5 \, f_{ywd,ef \, \text{rail no.}}) \]

\[ A_{sw} = (0.994 - 0.75 \times 0.569) \times 2496 \times 160 / (1.5 \times 313.5 \times 7) = 68.8 \, \text{mm}^2 \rightarrow \text{stud dia} = 10 \, \text{mm} (A = 78.54 \, \text{mm}^2) \]

Provide 7 No 10-5-240-890 Spacing: 125/160/160/160/160/125 (549.7 \, \text{mm}^2) 35 Studs total

Rail Layout

7 No 10-5-240-890 Spacing: 125/160/160/160/160/125 (549.7 \, \text{mm}^2) 35 Studs total
12. Example calculation – Corner condition

Data

Slab depth \( h = 275 \text{ mm} \)

Load \( V_{\text{ED}} = 215 \text{ kN} \) (load reduced to produce a working calculation following an amendment to the National Annex)

Cover = 30 mm (top and bottom)

Reinforcement T1 & T2 = H16 @ 150c/c

Compressive strength of concrete \( f_{\text{ck}} = 30 \text{ MPa} \)

a. Spiral/Circular Pattern.

Internal column \( \beta = 1.5 \) (unless advised otherwise by the Project Engineer).

\[
d = 275 - 30 - 16/2 - 16/2 = 229 \text{ mm}
\]

\[
d_y = 275 - 30 - 16/2 = 237 \text{ mm}
\]

\[
d_z = 275 - 30 - 16 - 16/2 = 221 \text{ mm}
\]

Shear at the column face

\[
\begin{align*}
\beta V_{\text{Ed}} &= 1.5 \times 215 = 322.5 \text{ kN} \\
\nu_{\text{Ed}} &= \beta V_{\text{Ed}} / (u_0 d) = 322.5 \times 1000 / (600 \times 229) = 2.347 \text{ MPa} \\
f_{\text{cd}} &= \alpha_{\text{cc}} f_{\text{ck}} / \gamma_c = 1 \times 30 / 1.5 = 20 \text{ MPa} \\
\nu_{\text{Rd,max}} &= 0.3 f_{\text{cd}} (1 - (f_{\text{ck}} / 250)) = 0.3 \times 20 (1 - (30 / 250)) = 5.28 \text{ MPa} \text{ or check if } \nu_{\text{Ed}} \leq \nu_{\text{Rd,max}} = 2.347 \text{ MPa} \leq 5.28 \text{ MPa} \text{ OK!}
\end{align*}
\]

Shear at control perimeter at 2d

\[
\begin{align*}
\nu_{\text{Ed}} &= 2 \times 300 + \pi \times (2 \times 229) / 2 = 1320 \text{ mm} \\
\text{(note: any holes within 6d need to be allowed for)}
\end{align*}
\]

\[
\beta V_{\text{Ed}} = 1.5 \times 215 = 322.5 \text{ kN}
\]

\[
\nu_{\text{Ed}} = \beta V_{\text{Ed}} / (u_1 d) = 322.5 \times 1000 / (1320 \times 229) = 1.067 \text{ MPa}
\]

Consider reinforcement over 300 + 3 \times 229 = 0.987 m width in both directions from centre of column.

Using H16 @ 150c/c both directions = 1340.41 mm\(^2\)/m

\[
\begin{align*}
\rho_l &= \sqrt{(A_{\text{shy}} / (b d_y)) \times A_{\text{shz}} / (b d_z)) = \sqrt{(1340.41 / 237 \times 1340.41 / 221) / 1000} = 0.00586 < 0.02
\end{align*}
\]

\[
\begin{align*}

\nu_{\text{Rd,c}} &= C_{\text{Rd,c}} k (100 \rho_l f_{\text{ck}})^{1/3} = 0.12 \times 1.935 (100 \times 0.00586 \times 30)^{1/3} = 0.604 \text{ MPa}
\end{align*}
\]

check if \( \nu_{\text{Rd,c}} \geq \nu_{\text{min}} = 0.604 \geq 0.516 \text{ Ok! (use largest value)}

check if \( \nu_{\text{Ed}} < 2 \nu_{\text{Rd,c}} = 1.067 > 0.604 \text{ Shear reinforcement required}

check \( \nu_{\text{Ed}} > 2 \nu_{\text{Rd,c}} = 1.067 > 1.208 \text{ Below } 2 \nu_{\text{Rd,c}} \text{ limitation}

\[
U_{\text{out required}} = \beta V_{\text{Ed}} / (\nu_{\text{Rd,c}} d) = 1.5 \times 215 \times 1000 / (0.604 \times 229) = 2331.6 \text{ mm (note: any holes within 6d need to be allowed for)}
\]

\[
\text{December 2009}
\]
Shearail Design Manual to EC2

Shearail Layout – Spiral/Circular Pattern

\[ 0.75d = 171.75 \text{ mm} \]

\[ 300/2 = 150 \text{ mm} \quad \text{therefore: position rail central about column face in each direction hence} \quad g_2 = 150 \text{ mm} \]

\[ U_{\text{out required}} = 2331.6 \text{ mm} \]

\[ e_1 = (300 - 150) = 150 \text{ mm} \]

\[ e_2 = (300 - 150) = 150 \text{ mm} \]

\[ e = \frac{(U_{\text{out}} - e_1 - e_2)}{3} = \frac{2331.6 - 150 - 150}{3} = 677.2 \text{ mm} \]

\[ g = 1.932e = 1.932 \times 677.2 = 1308.36 \text{ mm} \]

\[ g_1 = g - 1.553d - g_2 = 1308.36 - 150 - 150 = 908.3 \text{ mm} \]

\[ 1^\text{st} \text{ stud from column face} = 0.5d = 114.5 \text{ mm} \quad \text{say} 110 \text{ mm} \]

\[ \text{distance of} \ 1^\text{st} \text{ to last stud} = 807.3 - 110 = 797.3 \text{ mm} \quad \text{say} 5 \text{ @} 160 \text{ mm} < 0.75d = 171.75 \text{ mm} \]

\[ \text{Maximum stud distances on perimeter} \ (s_{1.5}) = 1.5d = 343.5 \text{ mm} \quad \text{&} \quad (s_{\text{last}}) = 2.0d = 458 \text{ mm} \]

\[ f_{\text{ywd,ef}} = 250 + 0.25 \times 229 = 307.25 \text{ N/mm}^2 \]

\[ f_{\text{ywd}} = \frac{f_y}{1.15} = \frac{500}{1.15} = 434.78 \text{ N/mm}^2 > 307.25 \text{ ok!} \]

\[ \text{Check standard spacing less then} \ 1.5d \text{ within} \ 2d \text{ from column face.} \]

\[ \text{Distance to} \ 1^\text{st} \text{ stud} = (150 / \cos 30^\circ) + 110 = 283 \text{ mm} \]

\[ \text{length to} \ 3^\text{rd} \text{ stud from column face} = 2 \times 160 + 283 = 603 \text{ mm} \]

\[ s_{1.3} = \sqrt{(603^2 + 603^2 - 2 \times 603 \times 603 \times \cos 30^\circ)} = 312 \text{ mm} < 1.5d \]

\[ \text{length to} \ 4^\text{th} \text{ stud from column face} = 3 \times 160 + 283 = 763 \text{ mm} \]

\[ s_{1.4} = \sqrt{(763^2 + 763^2 - 2 \times 763 \times 763 \times \cos 30^\circ)} = 395 \text{ mm} < 2.0d \]
length to 5th stud from column face = 4 x 160 + 283 = 923 mm

\[ s_{t5} = \sqrt{(923^2 + 923^2 - 2 \times 923 \times 923 \times \cos 30°)} = 478 \text{ mm} > 2.0d \]

(provide one splitter rail from the 5th stud)

**Shear at the control perimeter with reinforcement**

\[ A_{sw,\text{min}} = 0.08 \cdot s_t \cdot s_r \cdot \sqrt{f_{ck}} / (1.5 \cdot f_{yk}) = 0.08 \times 160 \times 404 \sqrt{30} / (1.5 \times 500) = 37.8 \text{ mm}^2 \]

\[ A_{sw} = (v_{Ed} - 0.75 \cdot v_{Rd,c}) \cdot u_1 \cdot s_r / (1.5 \cdot f_{ywd,ef} \cdot \text{rail no.}) \]

\[ A_{sw} = (1.067 - 0.75 \times 0.604) \times 1320 \times 160 / (1.5 \times 313.5 \times 4) = 68.94 \text{ mm}^2 \rightarrow \text{stud dia} = 10 \text{ mm} (A = 78.54 \text{ mm}^2) \]

**Rail Layout**


3 No 10-2-215-380 Spacing: 110/160/110 30 Studs total
Alternative Shearail Layout – Spiral/Circular Pattern

Try: 5 spurs

\[ A_{sw} = (1.067 \times 0.75 \times 0.604) \times 1320 \times 160 / (1.5 \times 313.5 \times 5) = \]

= 55.15 mm² → stud dia = 10 mm (A = 78.54 mm²)

Rail Layout


30 Studs total
Holes/Penetrations in the slab

If the shortest distance between the perimeter of the loaded area and the edge of the opening does not exceed 6d, that part of the control perimeter contained between two tangents drawn to the outline of the opening from the centre of the loaded area is considered to be ineffective, as quoted in clause 6.4.2 (3)

When \( l_1 \) is less than or equal to \( l_2 \)

When \( l_1 \) is greater than \( l_2 \)
When the hole is inside the stud arrangement the rails should be cut back and additional rails added either side of the hole.

Note: the studs shown in red cannot be used to calculate the steel area provided; this may result in larger diameter studs being required, unless full length splitter rails are used. The effected rail should be cut back to conform the spacing requirements.
14. Enhanced Stud Spacing

Rectangular / Square Columns.

When considering the results from the full scale testing, it is possible to increase the stud spacing to a maximum of 3.5d for studs outside of the 2d perimeter.

Using the standard set out geometry the maximum distance on the 8th stud is 3.366d which is within the 3.5d tested.

Provide intermediate rails as below, when more than 8 perimeters of reinforcement is required (8 studs on a rail).

If the standard rail/stud setting out pattern is not possible additional manual spacing checks will be required to maintain spacing rules:

- 1.5d between studs inside the 2d perimeter (U1)
- 3.5d between studs after the 2d perimeter.

Provide intermediate corner rails when more than 8 perimeters of reinforcement are required.

* Denotes: \( g_2 = 0.75d \) maximum with the first stud at 0.5d from the column face.

** Denotes: 0.5d may be reduced to 0.3d and the dimension \( g_2 \) can be increased 0.95d accordingly, to retain a similar geometry.
Enhanced Stud Spacing – Circular Columns.

Calculate the position of the perimeter $U_{out}$ or $U_{out,ef}$.

As normal, the rails spiral around the column at an angle between $30^\circ$ to $45^\circ$ giving equal lengths around $U_{out}$ for ‘$e$’, resulting with a layout of 8 to 12 main rails.

The same prefabricated rail can be used throughout the layout.

General rules for a Spiral/Circular Pattern

*position the last stud from column face* $= g_1$

minimum number of studs on a rail is 2

$U_{out} = \beta \ V_{Ed} / (n_{Rd,c} \ d)$

The number of rail spurs depends on the spacing rules inside 2d perimeter (less than or equal to 1.5d) and on the last stud spacing (less than or equal to 3.5d).

As $U_{out}$ is a polygon of equal sides: the number of sides = the number of main rail spurs.

$e_3 \leq 3.5d$ (without intermediate rails) or $e_3 \leq 7d$ (with intermediate rails)

$e = \frac{U_{out}}{\text{No. of spurs}}$

$e_3 = e - 2e_2$ where $e_2 = 1.5d \ Tan F^\circ$

$F^\circ = 180 / \text{No. of spurs}$ (sides or main spurs)

Therefore try 8 spurs (as a standard layout) increasing the number of spurs until $e_3 \leq 3.5d$ or $e_3 \leq 7d$ with intermediate rails.

$g = \frac{e}{\sin E^\circ} \times \sin ((180^\circ - E^\circ) / 2)$

where $E^\circ = 360 / \text{No. of spurs}$

$g_1 = g - g_2 - g_3$ where $g_2 = \text{column diameter} / 2$

$g_3 = 1.5d / \cos F^\circ$

$1^{st}$ stud from column face = 0.3d min. to 0.5d max. (less than or equal to 0.75d)
Final Spiral/Circular layout Pattern

* Denotes: \( g_2 = \text{Column diameter} / 2 \)

** Denotes: 0.5d may be reduced to 0.3d